Table 2 Contributions to the vertical velocity at $r = 0$

z	$\frac{\partial\phi_0}{\partial z}$	$\frac{\partial\phi_1}{\partial z}$	$\frac{\partial\phi_2}{\partial z}$
0.2	−0.200	0.064	−0.002
0.4	−0.400	0.112	0.012
0.6	−0.600	0.128	0.034
0.8	−0.800	0.096	0.062

is appreciable may be seen from the contribution of the first two perturbations given in Tables 1 and 2.

References

[1] Shaprio, A. H., *Compressible Fluid Flow* (Ronald Press Co., New York, 1953), Vol. I, Chap. 12, p. 365.

[2] Prandtl, L. and Tietjens, O. G., *Fundamentals of Hydro- and Aeromechanics* (Dover Publications, Inc., New York, 1957), Chap. X, p. 142.

Comparison of Stability Data Obtained in Free-Flight and Steady-State Facilities

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TESTS in gases with values of the specific heat ratio γ different from that of 1.400 are being performed with increasing frequency. Helium is useful to alleviate the gas liquefaction problem inherent in the use of air or carbon dioxide for hypersonic flow. Carbon dioxide is useful to simulate entry into planetary atmospheres, composed for the large part of carbon dioxide, and to simulate the imperfect gas effects characterized by a change in γ but is more subject to the problem of liquefaction than is air.

To circumvent this liquefaction problem at high Mach numbers, a free-flight method of testing has been evolved which allows the stability of a vehicle to be determined from an observation of its dynamic response along a short trajectory when fired through a test gas[1]. Reference 2 contains experimental stability results obtained in air, argon, helium, and carbon dioxide by this method which show that the stability of blunt, flared vehicles is greatly affected by the specific heat ratio γ of the test gas. (It should be noted that an inquiry into the source of these data disclosed that, whereas most of the data came from free-flight tests, part of the test data at $\gamma = \frac{5}{3}$ was obtained from sting mounted models.) The Reynolds number range of these tests varied from two to five million based on the stream conditions ahead of the model and the model length.

An identical model has been tested recently in the steady-state Langley 11-in. hypersonic tunnel in both air and helium at a Mach number of approximately 10.1 and a Reynolds number of approximately 0.6×10^6. Although this Reynolds number is approximately $\frac{1}{3}$ the lowest Reynolds number of Ref. 2, unpublished Langley data have shown that C_{m_α} is affected less than 25% by this Reynolds number variation. The ratio of these steady-state stability results is plotted on Fig. 1 together with the ratio of the faired data from Ref. 1. The agreement between these sets of data confirms the wide variation in C_{m_α} as a function of γ and lends credence to the accuracy of free-flight techniques.

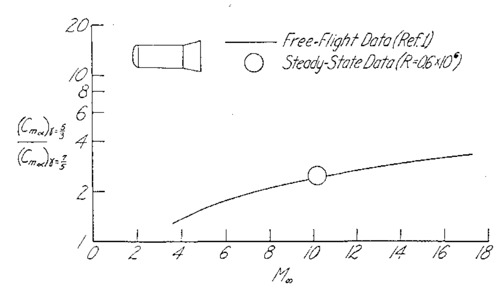

Fig. 1 Comparison of longitudinal stability parameter data from free-flight and steady-state techniques.

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References

[1] Seiff, A., "A free-flight wind tunnel for aerodynamic testing at hypersonic speeds," NACA Rept. 1222 (1955).

[2] Seiff, A., "Recent information on hypersonic flow fields," *Proceedings of the NASA-University Conference on the Science and Technology of Space Exploration*, NASA SP-11, Vol. 2, pp. 269–282 (November 1962).

Flame Stabilization in Laminar Boundary Layers

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Nomenclature

a	= rate of consumption of combustible component
C, C_1, C_2, C_3, C_4	= const
c_p	= heat capacity at constant pressure
D	= diffusion coefficient
e	= temperature ratio, $e = (T - T_0)/(T_\infty - T_0)$
f	= Blasius function, $f = f(\eta)$
K	= concentration ratio, $K = (k - k_0)/(k_\infty - k_0)$
k_i	= mass fraction of species i in mixture
p	= pressure
Pr	= Prandtl number, $Pr = c_p\mu/\lambda$
q	= reaction enthalpy per unit mass
R	= molar gas constant
r	= displacement ratio, $r = x/x^*$
Sc	= Schmidt number, $Sc = \mu/\rho D$
T	= temperature
U	= velocity in the streamwise direction in the Howarth plane
u	= velocity in the streamwise direction in the physical plane
V	= velocity in the normal direction in the Howarth plane
v	= velocity in the normal direction in the physical plane
X	= streamwise displacement in the Howarth plane
x	= streamwise displacement in the physical plane
Y	= normal displacement in the Howarth plane
y	= normal displacement in the physical plane
η	= Blasius variable, $\eta = Y(U_\infty/\nu_\infty X)^{1/2}$
ζ	= dimensionless variable, $\zeta = X/U_\infty\tau$
λ	= heat conduction coefficient
μ	= dynamic viscosity
ν	= kinematic viscosity
ξ	= characteristic stay-time, $\xi = X/U_\infty$
ρ	= density
τ	= reaction time factor

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Superscripts

0	=	conditions when there is no chemical reaction
1	=	first perturbation
*	=	refers to conditions at the flame front

Subscripts

0	=	conditions at the surface
∞	=	freestream conditions

A CHEMICALLY active gaseous mixture is considered as it flows over a heated flat plate. Of interest is the region where chemical reaction is initiated in the boundary layer and the resulting flame spreads to the freestream. In this case, the gas phase reaction accounts for most of the energy release, and the secondary catalytic effect of the solid surface is negligible.[1] In the analysis it is assumed that significant results may be obtained using the boundary-layer approximations, and this proves to be correct when the analytical results are compared with experimental ones. The Howarth transformations[2,3]

$$U = u \qquad V = \left\{\frac{\rho}{\rho_\infty}\right\} v + u \int_0^y \frac{\partial}{\partial x}\left\{\frac{\rho}{\rho_\infty}\right\} dy$$

$$X = x \qquad Y = \int_0^y \left\{\frac{\rho}{\rho_\infty}\right\} dy$$

and the usual assumption $\mu\rho = C\mu_\infty\rho_\infty$, and use of the dimensionless variables

$$e = (T - T_0)/(T_\infty - T_0) \qquad K = (k - k_0)/(k_\infty - k_0)$$

allow the writing of the governing equations in the following form:

Continuity Equation

$$(\partial U/\partial X) + (\partial V/\partial Y) = 0 \tag{1}$$

Momentum Equation

$$U(\partial U/\partial X) + V(\partial U/\partial Y) = \nu_\infty(\partial^2 U/\partial Y^2) \tag{2}$$

Energy Equation

$$U\frac{\partial e}{\partial X} + V\frac{\partial e}{\partial Y} = \left(\frac{\nu_\infty}{Pr}\right)\frac{\partial e^2}{\partial Y^2} + \frac{aq}{c_p(T - T_0)} \tag{3}$$

Species Equation

$$U\frac{\partial K}{\partial X} + V\frac{\partial K}{\partial Y} = \left(\frac{\nu_\infty}{Sc}\right)\frac{\partial^2 K}{\partial Y^2} - \frac{a}{\rho_\infty(k - k_0)} \tag{4}$$

The global equation of state $p = \rho RT$ has been used in the preceding equations.

The approximate boundary conditions are, for a flat plate at constant temperature,

$$\text{at } Y = 0\text{: } U = 0 \qquad V = 0 \qquad e = 0 \qquad K = 0 \tag{5}$$

$$\text{at } Y = \infty\text{: } U = U_\infty \qquad \partial V/\partial Y = 0 \qquad e = 1, \quad K = 1 \tag{6}$$

The continuity and the momentum equations [(1) and (2)] are solved using the Blasius method. The energy and species concentration equations [(3) and (4)] are solved using an iterative method. It is noted that a Blasius-type solution may be obtained for Eqs. (3) and (4) for the case of no chemical reaction, i.e., $a = 0$. Such solutions are obtained first and are denoted by $e°$ and $K°$, respectively. Then the method of successive approximations is used to obtain better approximations to solutions considering chemical reactions, i.e., $a \neq 0$.

Let $\eta = Y(U_\infty/X\nu_\infty)^{1/2}$ and let $f(\eta)$ satisfy $2f''' + ff'' = 0$, with the boundary conditions at $\eta = 0$: $f = 0$, $f' = 0$, at $\eta = \infty$: $f' = 1$. Then setting

$$e|_{a=0} = e^\circ(\eta)$$

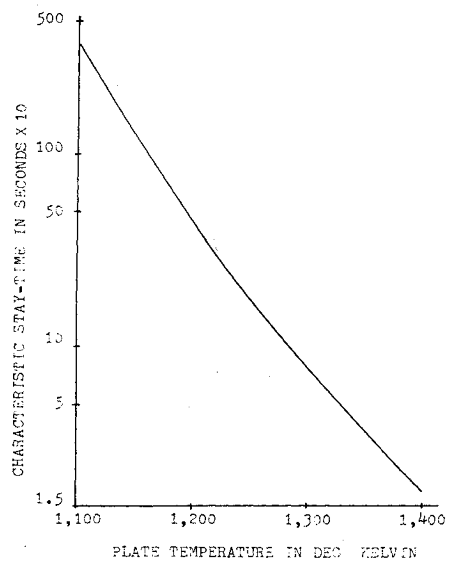

Fig. 1 Variation of characteristic stay-time with plate temperature.

Eq. (3) becomes

$$\left(\frac{2}{Pr}\right)\frac{d^2e^\circ}{d\eta^2} + f\frac{de^\circ}{d\eta} = 0$$

From this it follows that

$$e^\circ = C_1 + C_2\int_0^\eta \left[\exp\left(-\frac{\mathrm{Pr}}{2}\int_0^\eta f d\eta\right)\right] d\eta \tag{7}$$

Similarly, we have

$$K^\circ = C_3 + C_4\int_0^\eta \left[\exp\left(-\frac{Sc}{2}\int_0^\eta f d\eta\right)\right] d\eta \tag{8}$$

The expressions for $e°$ and $K°$ hold near the leading edge and up through the region where the chemical reaction is slight. In order to obtain successively better solutions, e and K are expressed in series form such as

$$e = e^\circ(\eta) + e^1(\eta)\zeta + e^2(\eta)\zeta^2 + \ldots$$

$$K = K^\circ(\eta) + K^1(\eta)\zeta + K^2(\eta)\zeta^2 + \ldots$$

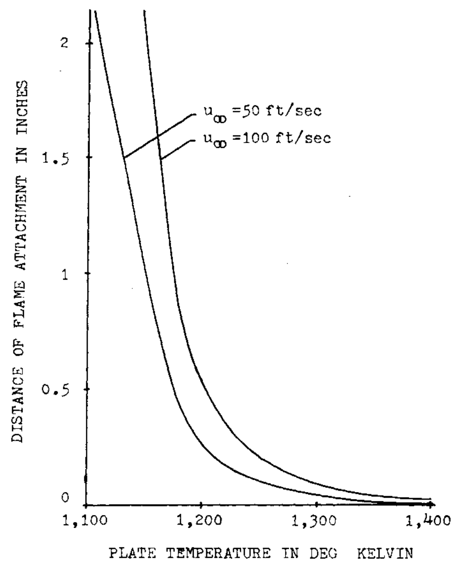

Fig. 2 Variation of the distance for flame attachment with plate temperature and freestream velocity.

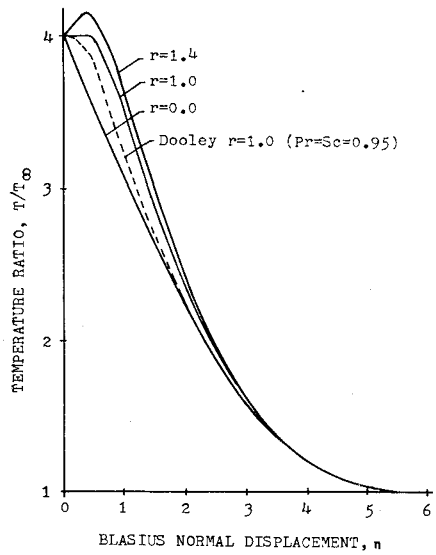

Fig. 3 Variation of the temperature ratio within the boundary layer at varying distances from the flame front.

where

$$\zeta = \frac{X}{U_\infty \tau}$$

and we obtain the following equation for e^1:

$$\left(\frac{1}{Pr}\right)\frac{d^2e^1}{d\eta^2} + \frac{f}{2}\frac{de^1}{d\eta} - f'e^1 = \frac{-qa^\circ}{c_p(T_\infty - T_0)} \tag{9}$$

Similarly, we have

$$\left(\frac{1}{Sc}\right)\frac{d^2K^1}{d\eta^2} + \frac{f}{2}\frac{dK^1}{d\eta} - f'K^1 = a^\circ \tag{10}$$

the boundary conditions being

$$\text{at } \eta = 0: \; e^1 = 0 \qquad K^1 = 0$$

$$\text{at } \eta = \infty: \; e^1 = 1 \qquad K^1 = 1$$

Equations (9) and (10) are readily integrated using numerical techniques on high-speed electronic computers. Higher-order approximations can also be obtained; however, these are not needed for the application considered below.

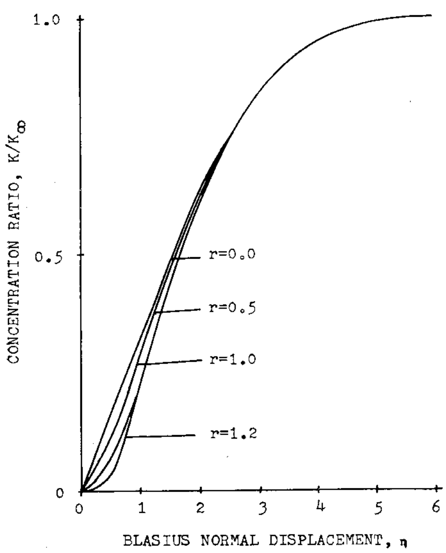

Fig. 4 Variation of the concentration ratio within the boundary layer at varying distances from the flame front.

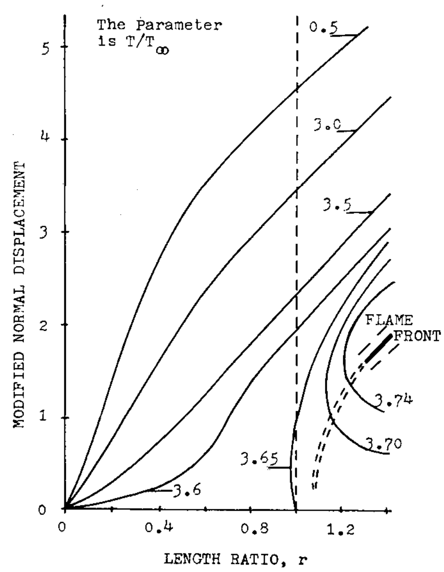

Fig. 5 Temperature profiles within the boundary layer.

Application and Conclusions

The present method was applied to the case of the decomposition of an azomethane stream flowing over a heated plate ($C_2H_6N_2 \rightarrow N_2 + C_2H_6$). This reaction has been studied by Sickman and Rice[4] who give its pertinent properties. An IBM 1620 digital computer was used in solving Eqs. (9) and (10), and the results are shown graphically in Figs. 1–5.

Figure 1 shows the exponential variation of the characteristic stay-time ξ with the plate temperature. This type of variation is supported by the observations of Ziemer and Cambel[5] and Dooley,[2] and it is due to the dominance of the reaction term in Eqs. (3) and (4).

Figure 2 shows the variation of x^* with the plate temperature. Here the freestream velocity enters as a parameter, and the exponential behavior is again evident.

In Figure 3 we see the variation of the temperature within the boundary layer. The parameter $r = x/x^*$ is of importance. It is seen that the temperature profile changes from a typical Blasius profile for $r = 0$ to one with persistently higher temperatures as r increases. The result of Dooley's solution[2] (Prandtl number equal to the Schmidt number with a value of 0.95) is also shown. In this paper, the values $Pr = 0.91$ and $Sc = 1.0$ were used, and the significant quantitative differences between the two solutions are evident.

Figure 4 shows the variation of the relative concentration of undissociated azomethane vs the Blasius coordinate η. These curves behave in a manner similar to the temperature profiles shown in Fig. 3.

Figure 5 shows the temperature profiles in the r-y plane. The flame front corresponds to a local maximum in temperature and can be readily located. Outside the boundary layer it is seen that the flame is in the shape of an inclined plane. For smaller values of y, the flame front tends to bend towards the plate till it becomes practically normal to it. This is the type of behavior of the flame front actually recorded in photographs of reacting boundary layers by several observers.[5,6]

The advantage of the present solution is that it allows the treatment of chemical reactions with arbitrary orders of reaction and of cases where the Prandtl number does not equal the Schmidt number. Moreover, the present approach can easily be extended to flows with pressure gradients, which have been found[6] to influence strongly the stability properties of the anchored flame.

References

[1] Valentin, P., "Convection avec reaction chimique," Ann. Phys. 6, 271–329 (1961).

[2] Dooley, D., "Ignition in the laminar boundary layer of a heated plate," *1957 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1957), pp. 321–342.

[3] Toong, T. Y., "Ignition and combustion in a laminar boundary layer over a hot surface," *Sixth Symposium on Combustion* (Reinhold Publishing Corp., New York, 1957), pp. 532–540.

[4] Sickman, D. V. and Rice, O. K., "Studies on the decomposition of azomethane," J. Chem. Phys. **4**, 242–251 (1936).

[5] Ziemer, R. W. and Cambel, A., "Flame stabilization in the boundary layer of heated plates," Jet Propulsion, **28**, 592–599 (1958).

[6] Wu, W. S. and Toong, T. Y., "Further study on flame stabilization in a boundary layer: A mechanism of flame oscillations," *Ninth Symposium on Combustion* (Reinhold Publishing Corp., New York, 1963), pp. 49–58.

Accurate Values of the Exponent Governing Potential Flow about Semi-Infinite Cones

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It is well known that, for axisymmetric, inviscid, and incompressible flow about a semi-infinite cone, the velocity V on the surface of the cone varies with the distance s from the vertex in the following manner:

$$V = Cs^m \tag{1}$$

where C is simply a scaling constant. The flow is thus completely characterized by the exponent m, which is a function of Θ, the semivertex angle of the cone. Tabulated values of m do not appear to be readily available in the literature. Reference 1 gives a small graph of m vs Θ and also lists three references, including the original work of Ref. 2. However, none of the three is contained in ordinary technical libraries.

The equation relating m and Θ is

$$P_{m+1}'(-\cos\Theta) = 0 \tag{2}$$

where the prime denotes differentiation and the function P_{m+1} is the Legendre function that remains finite when its argument equals unity for all values of its order $m + 1$ (see Ref. 3). Using the series expansion of P_{m+1}, this equation was solved numerically for m at values of Θ ranging from 0° to 180° by 5° increments. The results are given in Table 1.

Table 1 Values of the exponent governing potential flow about semi-infinite cones

Θ, deg	m	Θ, deg	$1/m$
0	0.0000000	90	1.0000000
5	0.0037441	95	0.8779641
10	0.0145329	100	0.7715075
15	0.0316314	105	0.6779398
20	0.0544316	110	0.5951432
25	0.0825162	115	0.5214293
30	0.1156458	120	0.4554368
35	0.1537334	125	0.3960580
40	0.1968232	130	0.3423826
45	0.2450773	135	0.2936569
50	0.2987690	140	0.2492515
55	0.3582834	145	0.2086375
60	0.4241237	150	0.1713675
65	0.4969244	155	0.1370604
70	0.5774709	160	0.1053903
75	0.6667277	165	0.0760764
80	0.7658769	170	0.0488761
85	0.8763705	175	0.0235785
90	1.0000000	180	0.0000000

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References

[1] Rosenhead, L. (ed.), *Fluid Motion Memoirs. Laminar Boundary Layers* (Oxford University Press, Oxford, England, 1963), p. 429.

[2] Leuteritz, R. and Mangler, W., "Die symmetrische Potentialströmung gegen einen Kreiskegel," Untersuch. Mitt. Deut. Luftfahrtforsch. **3226** (1945).

[3] Hildebrand, F. B., *Advanced Calculus for Engineers* (Prentice Hall, Inc., New York, 1949), p. 177.

Variable Collision Frequency Effects on Hall-Current Accelerator Characteristics

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Nomenclature

B = magnetic induction
b = ion-slip parameter
E = electric field
e = charge on electron
J = current density
L = reference length
M = Mach number
m = mass flow
m_a = atom mass
N = interaction parameter
P = pressure
S = parameter defined by Eq. (4)
T = temperature
V = velocity
z' = dimensionless axial coordinate = z/L
α = degree of ionization
β = Hall parameter defined by Eq. (5)
γ = specific heat ratio
η = energy conversion efficiency
ρ = mass density
σ = electrical conductivity
τ = time between collisions
$\bar{\tau}_e$ = mean electron collision time
ϕ = voltage difference between inlet and exit
ω = cyclotron frequency

Subscripts

ei = electron-ion collision
ea = electron-atom collision
ia = ion-atom collision
0 = accelerator inlet
z = axial direction
θ = azimuthal direction

One-dimensional magnetogasdynamic (MGD) analyses of the coaxial Hall-current accelerator have recently been presented by Brandmaier, Durand, Gourdine, and Rubel[1] and by Cann and Marlotte.[2] Each considered the steady continuum flow of an ideal, electrically neutral, slightly ionized, three-species gas mixture through a narrow constant

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